Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

of the problem is based upon the definition of the probability of an event which is the ***total number of favorable cases divided by the total number of cases.***

In this problem, the favorable cases have been assumed to be proportional to the length of the arc $\phi \times PE$ and the total number of cases proportional to the length of the semi-circumference $\pi \times PE$.

Hence, the chance that the man crosses the pond for any particular value of ϕ, x and θ being constant, is

$$C_1 = \frac{\phi \times PE}{\pi \times PE} = \phi/\pi = \frac{\sin^{-1}[z \div (R^2 + 2x^2 + 2Rx\cos\theta)]}{\pi} = \frac{f(z)}{\pi}.$$

Now, all values of $f(z)$ between the limits $z=0$ and $z=R-x$ must be considered. Let

$$h = \frac{0+R-x}{n} = \frac{R-x}{n}. \quad \text{Then,} \sum_{z=0}^{z=R-x} f(z) = f(h) + f(2h) + f(3h) + \ldots. + f(nh).$$

The chance that $f(z)$ has the value of any particular term as $f(kh)$ of this series is $1/n$, or $h/(R-x)$.

Hence, the chance that the man crosses the pond is the product of the chance that $f(kh)$ is the value of $f(z)$.multiplied by $f(kh)/\pi = f(kh)/[\pi(R-x)]$.

Hence, the chance that the man crosses the pond for all values of $f(z)$ between the limits $z=0$ and $z=R-x$, x and θ being constant, is

$$C_2 = \frac{f(h)+f(2h)h+f(3h)h+\ldots+f(nh)h}{\pi(R-x)} = \frac{\int_0^{R-x} f(z)dz}{\pi(R-x)} = F(x).$$

$F(x)$ is the chance of the man crossing the pond when the center of the pond is at a certain fixed point. But the center of the pond may be any point in the field. The chance that the center of the pond falls on any point between two concentric circles whose radii are x and $x+dx$ is $\pi x dx/\pi R^2 = 2xdx/R^2$.

Hence, the chance of the man crossing the pond is $(2xdx/R^2)F(x)$. and for all values of x between $x=0$ to $x=R$ the chance is

$$C_3 = \int_0^R \frac{2xdx}{R^2} F(x) = \int_0^R \frac{2xdx}{R^2} \int_0^{R-x} \frac{f(z)dz}{\pi(R-x)} = \Psi(\theta).$$

The chance that θ has any value between θ and $\theta + d\theta$ is $d\theta/\pi$.

Hence, the chance that the man crosses the pond for all values of $\psi(\theta)$ between $\theta=0$ to θ is

$$C_4 = \int_0^\pi \frac{d\theta}{\pi} \Psi(\theta) = \int_0^\pi \frac{d\theta}{\pi} \int_0^R \frac{2xdx}{R^2} F(x) = \int_0^\pi \frac{d\theta}{\pi} \int_0^R \frac{2xdx}{R^2} \int_0^{R-x} \frac{f(z)dz}{\pi(R-x)}.$$

Thus it appears that the first method when made to obey strictly the definition of probability leads to the same form for computation as the second. The first two solutions of this problem as published in the December Number, of Vol. VII, of the MONTHLY, are, therefore, incorrect.

Dr. Martin informs me that Professor Seitz sent him a solution of this problem and obtained the same result as that given in the two solutions previously published. Professor Seitz, in his day, solved a great many very difficult probability problems, the solutions of very few of which have since been found erronous, if the laws of distribution assumed by him be granted.

The above solution of the "pond problem" has the sanction of the ablest living mathematicians in this country.

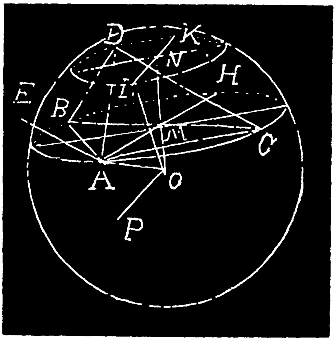
This point, viz., that the total number of events are not to be obtained in such problems by writing for the denominator a certain set of definite integrals, should not be overlooked in solving probability problems of this particular in the future.

130. Proposed by **LON C. WALKER, A. M.**, Graduate Student, Leland Stanford University, Cal.

Four points are taken at random on the surface of a given sphere; show that the average volume of a tetrahedron formed by the planes passing through the the points taken three and three, is 1-35 of the volume of the given sphere.

I. **Solution by the PROPOSER.**

Choose A, B, C, D as the four random points; O the center of the given sphere with radius r; $ABFE$ a great circle through A, B; ABC a small circle through A, B, C with center S; DGF a small circle through D parallel to ABC with center P; M the middle point of AB.



Put $OP=x$, $AS=r_1$, $\angle AOB=\theta$, $\angle OMS=\phi$, $\angle CAB=\psi$, $\angle SAM=\psi_1$. Then we have

$AM=r\sin\frac{1}{2}\theta=r_1\cos\psi_1$,

$SM=r\cos\frac{1}{2}\theta\cos\phi=r_1\sin\psi_1$,

$OS=r\cos\frac{1}{2}\theta\sin\phi$,

$AC=2r_1\cos(\psi-\psi_1)$,

$PD=\sqrt{(r^2-x^2)}$,

$r_1=r(\sin^2\frac{1}{2}\theta+\cos^2\frac{1}{2}\theta\cos^2\phi)^{\frac{1}{2}}$, area $ABC=2rr_1\sin\frac{1}{2}\theta\sin\psi\cos(\psi-\psi_1)$,

volume of tetrahedron $D-ABC=\frac{1}{3}SP.\text{area}ABC=\frac{2}{3}rr_1(x+r\cos\frac{1}{2}\theta\sin\phi)\sin\frac{1}{2}\theta\times\sin\theta\cos(\psi-\psi_1)$. Hence we have for the required average volume

$$V=\frac{1}{(4\pi r^2)^8}\int_0^\pi\int_0^\pi\left[\int_0^{\frac{1}{2}\pi+\psi_1}\left(\int_{-r\cos\frac{1}{2}\theta\sin\phi}^{r}\frac{2}{3}rr_1(x+r\cos\tfrac{1}{2}\theta\sin\varphi)\sin\tfrac{1}{2}\theta\sin\psi\right.\right.$$

$$\cos(\psi-\psi_1).2\pi r dx$$

$$\left.+\int_{r\cos\frac{1}{2}\theta\sin\phi}^{r}\frac{2}{3}rr_1(x-r\cos\tfrac{1}{2}\theta\sin\phi)\sin\tfrac{1}{2}\theta\sin\psi\cos(\psi-\psi_1).2\pi r dx\right)4rr_1\sin\psi\cos(\psi-\psi_1)d\psi$$

$$+\int_0^{\frac{1}{2}\pi-\psi_1}\left(\int_{-r\cos\frac{1}{2}\theta\sin\phi}^{r}\frac{2}{3}rr_1(x+r\text{cys}\tfrac{1}{2}\theta\sin\phi)\sin\tfrac{1}{2}\theta\sin\psi\cos(\psi+\psi_1).2\pi r dx\right.$$

$$\left.\left.+\int_{r\cos\frac{1}{2}\theta\sin\phi}^{r}\frac{2}{3}rr_1(x-r\cos\tfrac{1}{2}\theta\sin\phi)\sin\tfrac{1}{2}\theta\sin\psi\cos(\psi+\psi_1).2\pi r dx\right)4rr_1\sin\psi\cos(\psi+\psi_1)\right]d\phi$$

$$\times 2\pi r^2\sin\theta d\theta$$

$$=\frac{r}{3}\int_0^\pi\int_0^\pi\left[\int_0^{\frac{1}{2}\pi+\psi_1}r_1\sin^2\psi\cos^2(\psi-\psi_1)d\psi+\int_0^{\frac{1}{2}\pi+\psi_1}r_1^2\sin^2\psi\cos^2(\psi+\psi_1)d\psi\right]$$

$$\sin^2\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta(1+\cos^2\tfrac{1}{2}\theta\sin^2\phi)d\theta d\phi$$

$$=\frac{r^3}{24}\int_0^\pi\int_0^\pi(\sin^2\tfrac{1}{2}\theta+3\cos^2\tfrac{1}{2}\theta\cos^2\phi)(1+\cos^2\tfrac{1}{2}\theta\sin^2\phi)\sin^2\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta d\theta d\phi$$

$$=\frac{r^3}{24}\int_0^\pi(\tfrac{32}{35}+\tfrac{12}{35}\cos2\phi-\tfrac{2}{35}\cos4\phi)d\phi=\tfrac{4}{105}\pi r^3=\tfrac{1}{35}\text{ of the volume of given sphere.}$$